Neural Network Methods for Construction of Sociodynamic Models Hierarchy

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Abstract. The article includes the following modern approaches to modelling sociodynamic processes: Kolmogorov equation system for Markov process with a discrete set of states, Fokker-Planck equation, multiagent systems, etc. As an example, one demographic task of predicting is solved. We compare the simplest neural network approach with an approach based on a special evolutionary model. The article also justifies applying the neural network modelling for producing solutions to the above mentioned equations, determination of their coefficients on the basis of observations and making more precise models, including the dependence of human behaviour on a psychological type. A possibility of making models more precise as new data come in has been discussed.

Keywords: Sociodynamics · Artificial neural networks · Markov processes · Kolmogorov equation · Fokker-Planck equation · Hierarchical systems · Multiagent systems

1 Introduction

Over the past years the problem of predictive models for sociodynamic processes has been gaining growing importance. Thus, a decision was adopted in the European Union to build a global system in 2013–2020 for prediction of social, economic, and environmental occurrences in their interrelation [1]. A similar task was stated at the Plenary Session of 16th St.-Petersburg International Economic Forum [2].

In order to solve this problem it is necessary to construct a hierarchy of interrelated mathematical models in economics, sociology, politology, etc. that can approach in their accuracy and predictive power to physical models. The statistical physics performs transition from molecular dynamics to equations for continuous medium. When developing sociodynamic models it is expedient

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to take initial data, as individual behaviour, including psychological and other similar factors, and then proceed to economical, social, mental (mems), etc. interrelating fields.

The main scientific problem, which must be dealt with when creating this kind of mathematical and information models, consists in indispensable transition from opinion-based and semi-empirical approach to application of modern mathematic simulation methods, justified planning, and optimal management typical of science and engineering.

It is essential to select classes of mathematic models and a method for their application. The requirements for these models can be stated as follows:

- Possibility to construct a model with any degree of complexity model using standard rules and standard elements
- Robustness of model against errors in initial data and a priori assumptions
- Possibility to deal with contradictory assumptions
- Possibility to adjust the model after receiving new data

All these requirements are met in neural network models [3,4].

2 Modern Approaches to Sociodynamic system modelling

Designing a comprehensive programme of developing a particular region, for example, the Arctic zone of the Russian Federation, involves tackling a great number of economic, environmental and social problems. Major mineral extraction projects are not only supposed to be cost-effective, but also take into account their influence on the environment and the population as well as the consequences for the Russian economy. As a result, interaction between experts in different fields is necessary. These experts should formalize their knowledge into uniform mathematical models in order to attain mutual understanding.

A system of Kolmogorov differential equations for Markov process with a discrete set of states was proposed by [5] in the capacity of such models. Let us describe this model in simplest case, when system state is assigned by vector $\mathbf{n}$ with integer coordinates $\{n_i\}$. The examples of such vectors are the sizes of particular national, social, etc. groups of population, the numbers of companies involved in different types of business and varying in size. Let $P(\mathbf{n}, t)$ be a probability of being in state $\mathbf{n}$ at time point t. We denominate as $\mathbf{n}_{ji}$ a vector, which differs from $\mathbf{n}$ by increase of coordinate j and decrease of coordinate i by 1. Then $\mathbf{n}_{i+}$ is a vector with coordinate i increased by 1, and $\mathbf{n}_i$ is a vector with coordinate i decreased by 1. Then "Main Equation" will look as follows:

$$\begin{aligned}\frac{dP(\mathbf{n},t)}{dt} = &\sum_{i\neq j} w_{ij}(\mathbf{n}_{ji},t)P(\mathbf{n}_{ji},t) - \sum_{i\neq j} w_{ji}(\mathbf{n},t)P(\mathbf{n},t) + \\ &\sum_i w_{i+}(\mathbf{n}_{i-},t)P(\mathbf{n}_{i-},t) - \sum_i w_{i+}(\mathbf{n},t)P(\mathbf{n},t) + \\ &\sum_i w_{i-}(\mathbf{n}_{i+},t)P(\mathbf{n}_{i+},t) - \sum_i w_{i-}(\mathbf{n},t)P(\mathbf{n},t), \qquad (1)\end{aligned}$$

where

$$w_{ij}(\mathbf{n},t) = \left.\frac{\partial P(\mathbf{n}_{ij},\tau|\mathbf{n},t)}{\partial\tau}\right|_{\tau=t},\ w_{i\pm}(\mathbf{n},t) = \left.\frac{\partial P(\mathbf{n}_{i\pm},\tau|\mathbf{n},t)}{\partial\tau}\right|_{\tau=t}$$

are intensities of transitions from the current state to the state with an increased or decreased coordinate i.

If the process under consideration does not involve transitions $\mathbf{n}_{i+}$ and $\mathbf{n}_{i-}$ (if the demographic dynamics is considered, it means that there is neither natality, nor mortality, or, to be precise, they are disregarded in comparison with the migratory processes) the Eq. (1) looks simpler:

$$\frac{dP(\mathbf{n},t)}{dt} = \sum_{i\neq j} w_{ij}(\mathbf{n}_{ji},t)P(\mathbf{n}_{ji},t) - \sum_{i\neq j} w_{ji}(\mathbf{n},t)P(\mathbf{n},t). \quad (2)$$

Further analysis requires assumptions with regard to intensities of transitions $w_{ij}(\mathbf{n},t)$. Different variants are considered by [5], e.g.

$$w_{ij}(\mathbf{n},t) = \mu_{ij}n_j \exp[u_i(\mathbf{n}_{ij},t) - u_j(\mathbf{n},t)], \quad (3)$$

where u_i is a utility function of state i. This function is significant because its increase raises the probability of transition into state i and decreases the probability of reverse transition. The utility function is assumed to be linear relative to $\mathbf{n}$. Different variants of dynamics in the system under examination are considered further (in [5]), according to Eq. (1) or (2), and depending on ratio of coefficients in this function. The case of quadratic dependence of utility function on $\mathbf{n}$ is examined by [6]. On the basis of dynamics observed in real systems, coefficients of these dependencies are estimated that may allow generating prediction for the future. In addition, dependence of these coefficients on other factors is examined (e.g. dependence of migration properties on social and economic factors, which is also supposed to be linear). Evidently, real dependencies are usually non-linear; so, if linear approximation has poor accuracy, search of non-linear dependencies should be performed. For the above reasons, neural network functions are suggested as desired non-linear ones.

In [5], for the sake of simplifying the analysis, examination of stationary case $w_{ij}(\mathbf{n},t) = w_{ij}(\mathbf{n})$ was recommended, and proceeding to quasi-averages, transition rates being averaged according to probabilities. Then (2) is changed by equation system as:

$$\frac{d\hat{n}_i}{dt} = \sum_{i\neq j} w_{ij}(\hat{\mathbf{n}}) - \sum_{i\neq j} w_{ji}(\hat{\mathbf{n}}). \quad (4)$$

Besides Markov processes with a discrete set of states, processes with a continuous set of states are examined by [6]. Conditional distribution density $f(\mathbf{y},\tau|\mathbf{x},t)$ in such processes complies to the equation system:

$$\begin{cases} \dfrac{\partial f}{\partial t} + \sum\limits_{i=1}^{n} a_i \dfrac{\partial f}{\partial x_i} + \dfrac{1}{2} \sum\limits_{i,j=1}^{n} b_{ij} \dfrac{\partial^2 f}{\partial x_i \partial x_j} = 0; \\ \dfrac{\partial f}{\partial \tau} + \sum\limits_{i=1}^{n} \dfrac{\partial (a_i f)}{\partial y_i} - \dfrac{1}{2} \sum\limits_{i,j=1}^{n} \dfrac{\partial^2 (b_{ij} f)}{\partial y_i \partial y_j} = 0, \end{cases} \tag{5}$$

where

$$a_i = \lim_{\tau \to t} \frac{1}{\tau - t} \mathrm{M}[Y_i - X_i | \mathbf{X} = \mathbf{x}],\ b_{ij} = \lim_{\tau \to t} \frac{1}{\tau - t} \mathrm{M}[(Y_i - X_i)(Y_j - X_j) | \mathbf{X} = \mathbf{x}].$$

In particular, these equations are satisfied with solutions of differential equations system

$$\frac{dU_i}{dt} = a_i(U_1, \ldots, U_n, t) + \sum_{m=1}^{n} g_{im}(U_1, \ldots, U_n, t) \xi_m(t),\ b_{ij} = n g_i g_j,$$

where $\xi_m(t)$ are mutually independent, and represent a white noise.

In one-dimensional case these equations are reduced to

$$\frac{\partial f}{\partial t} + a \frac{\partial f}{\partial x} + \frac{1}{2} b \frac{\partial^2 f}{\partial x^2} = 0;\ \frac{\partial f}{\partial \tau} + \frac{\partial (af)}{\partial y} - \frac{1}{2} \frac{\partial^2 (bf)}{\partial y^2} = 0. \tag{6}$$

The second equation is commonly called a Fokker-Planck equation. The corresponding differential equation is the following

$$\frac{\partial U}{\partial t} = a(U, t) + g(U, t) \xi(t),\ b = g^2. \tag{7}$$

The problem of identifying a and b functions by real data arises again in these models. Besides, in order to solve these particular equations it is necessary to know the initial and boundary conditions. Tendency to zero on the infinity seems to be appropriate as boundary conditions, but it does not correspond to the practical problems, in which it is known that all the components x or some of them are not negative. Initial conditions are even more difficult. Commonly the results of monitoring the process are known instead of them, which means that the classical methods of solving mathematical physics problems, such as finite differences method, finite element method, etc., cannot be applied. The methods set forth in the scholarly works [3,4] allow formulating approximate values (5), (6) taking into consideration all the information available.

Using data on the distribution of the population of Russia at the $m = 14$ age groups for 2010–2014, decide to test the prediction task. Data for the first three years are used to construct an evolutionary neural network model, in the last two – to check the results of the projection of this model.

The first approach does not use Fokker-Planck equation. The population distribution by ages is looking as an radial basis function (RBF) neural network approximation in the form

$$u_1(x, t, \mathbf{a}) = \sum_{j=1}^{n} a_{j1} \exp\big(-a_{j2}(x - a_{j3})^2\big) \exp\big(-a_{j4}(t - a_{j5})^2\big), \tag{8}$$

where $n = 10$ is the number of neurons. The variable x is the age interval, the variable t is time, in our case, a year; $\mathbf{a} = \{a_{ji}\}$ is the matrix of neural network weights. The network weights are determined by the minimization of the error functional, built on the basis of available data. Here, it has the form

$$J = \sum_{k=1}^{3} \sum_{i=1}^{m} \left(u_1(x_i, t_k, \mathbf{a}) - n_{ki}\right)^2, \tag{9}$$

where n_{ki} is the normalized ($\sum_{i=1}^{m} n_{ki} = 1$) quantity index of the age group i in year t_k.

In the second approach we take into account the evolving nature of the required distribution function. As the basis functions neural network uses the fundamental solution of the Fokker-Planck equation [7]. The distribution function is sought as neural network outputs in the form

$$u_2(x, t, \mathbf{b}) = \sum_{j=1}^{n} \frac{b_{j1} b_{j3}}{\sqrt{t + b_{j2}^2}} \exp \frac{-b_{j3}^2 (x - b_{j4} + b_{j5}(t + b_{j2}^2))^2}{t + b_{j2}^2}, \tag{10}$$

where $n = 10$ is the number of neurons, weight matrix $\mathbf{b} = \{b_{ji}\}$ components are determined by the minimization of the error functional similar to the functional (9).

The graphs of neural network approximate models of population distributions in 2010 year are shown at Fig. 1.

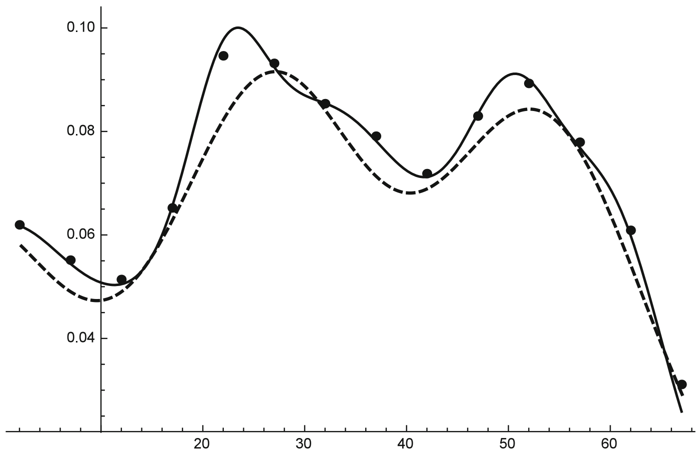

Fig. 1. Neural network approximate models for population distribution, 2010. Dots are real data, dashed line is the RBF-model, solid line is the model based on the Fokker-Planck equation

As we can see, neural network approximation description of the data in both models is good enough. These models predictions for 2014 are shown at the Fig. 2.

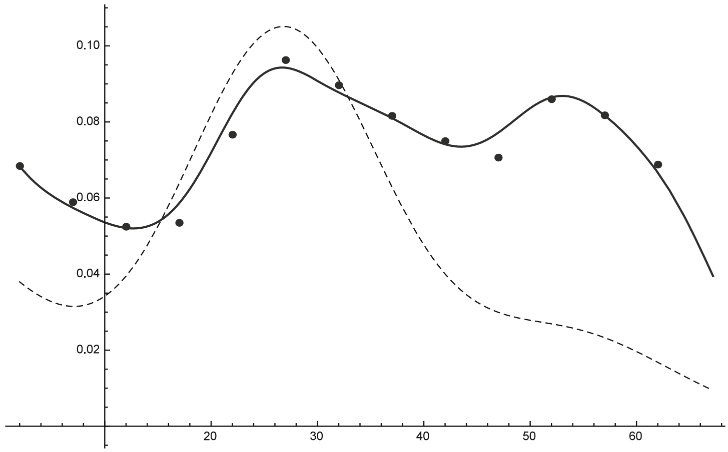

Fig. 2. Neural network models predictions for population distribution, 2014. Dots are real data, dashed line is the RBF-model prediction, solid line is the prediction of the model based on the Fokker-Planck equation

Obviously, the prediction of the model based on the Fokker-Planck equation is precise enough, while the RBF-model does not catch even the character the predicted distribution.

It is necessary to point out that the approach [3] to making stable neural network models on the basis of dissimilar data including differential data implies selecting not only free parameters, but also the structure of the model. It is particularly vital for sociodynamic models given the complexity of the object which is modelled. We can account additional conditions in the neural network models by adding to the error functional the corresponding terms.

For specific algorithms of this kind refer to [3] and the source literature quoted there.

3 Activity and Memory as Human Element

Besides their hierarchical structure, real engineering and socio-economic systems are characterised by activity and memory due to the human element. In order to optimise their operation, mathematical models of these phenomena are necessary to develop, as well as value scales, collective behaviour, etc. connected with them. Neuronal nets were already applied [8] for more simple models of such occurrences. It is advisable to combine this approach with methods for structural and parametrical identification of such systems on the basis of observed parameters [3,4], and management of such systems, including value scale management. For modeling we apply multiagent systems.

Particularly, the dynamics of the agent is assumed to follow minimisation task for functionals hierarchy, which may be interpreted as a value scale. At that, dynamics of the agent can be managed through changing internal parameters, adjusting external conditions (optimised by functionals) and rebuilding the value system (the position of the functional on a corresponding scale), eliminating some functionals and inserting other ones (that can optimise functionals of ambient system or managing the subject of the respective level).

If the agent simulates a human, particular dependencies are constructed based upon psychical features and specific interaction with environment.

Personality type 1: concentrated on model of reality as a whole, and on long-term prediction of its dynamics. Accuracy of model and its predictions makes a great contribution to quality functional. Activity is mainly directed at improvement of the model and realisation of desired development scenario.

Personality type 2: differs from Personality type 1 by substitution of reality as a whole for the nearest social environment.

Personality type 3: differs from Personality type 2 by obtaining quality functional evaluation from the environment. Need in adjusting activity according to variable functionals requires proceeding from long-time prediction to short-time ones.

Personality type 4: differs from Personality type 3 by obtaining not only quality evaluation, but also reality models from environment. High intensity of information exchange leads to reducing quantity of social contacts. Differences in models being obtained require significant efforts for their concordance and building a consistent picture. Main part in quality functional belongs to evaluation of one's own place in foreign reality models.

Personality type 5: differs by obtaining reality models from society as a whole that leads to reducing information exchange with the nearest environment, and long process of comparison between reality models and reality as such. Decrease of this discrepancy contributes the greatest part into desired change of quality functional.

Personality type 6: differs from Personality type 5 by striving for influence on reality as a whole, in order to bring it to concordance with one's internal model. Awareness of difficulty to make such influence alone by oneself leads to directing either at local possible changes or participation in great communities able to making global changes.

Personality type 7: differs from Personality type 6 by substitution of influence on reality for searching of own optimal place in reality, including optimal environment. Rapid changes in ambient reality lead to employing short-time predictions.

Personality type 8: differs from Personality type 7 by substitution of movement through reality for organisation of reality according to one's internal model and short-time predictions. For such organisation, social opportunities are actively employed, first of all, the nearest environment and accessible levels of social hierarchy.

Personality type 9: differs from Personality type 8 by substitution of changing ambient reality for changing one's own internal model according to ambient reality and opinions in the nearest environment. Such adjustment requires long concordance of different sub-models that leads to reduction of activity due to duration of such process.

4 Conclusion

Creating sociodynamics intellectual management systems is a problem the immediate future. We have presented an overview of the models of such systems. The analysis presented demonstrates the desirability of using the neural network technologies for modeling sociodynamics. We are confident that in the coming years will be to create a real system of prediction and control using the above approach.

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