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Mathematical Model of the Controlled Sustainable Development of the Region

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A new approach to regional management is based on an inverse solution, which reveals a constructive capability of applying "management by objectives" methods and models to guarantee the achievement of a regional management's objective. At the level of model implementation, the government's task is, based on special techniques, to develop and implement corresponding factors in a system of nonlinear equations so as to eventually achieve a management objective. The model helps to build the region's vital processes and guarantees achieving its development indicators on the basis of adequate management impact. The proposed concept is the fundamental condition for sustainable socio-economic development of a region in the interest of people, economy, and favorable environment. It is based on a synthesized model of government administration. The paper identifies the conditions for guaranteed sustainable management of a region based on a new dynamic model.

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1. Introduction

Government administration is practical implementation of appropriate procedure-based managerial decisions by governmental bodies and their officials. During the routine decision-making process, a civil servant would always like to know to what extent the result of his or her managerial activities lives up to his or her expectations. Correct conclusions as to the results of managerial activities can only be made based on a formal axiomatic method.

Managing a region, just like any other type of management, is based on the results of modeling the processes of this region's socio-economic development within the framework of a chosen management concept [1]. In system engineering, there are only two approaches to developing systems [2, 3]:

1. Development of an analysis-based system (development of an analysis-based model).
2. Development of a synthesis-based system (synthesis of a model) [4].

Historically and traditionally, socio-economic systems are controlled with the help of analysis-based models, where regularities in the researched field cannot be fully considered in the modeling process [5, 6]. A rather high level of risk incurred by the use of analysis-based models results in the managerial outcomes not fully matching decision-maker's expectations [7],), i.e. their use does not quite ensure the conditions for the guaranteed achievement of a government administration objective. When developing a methodological support at the strategic planning

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stage, one should consider the scientifically proven concepts of socio-economic development risks.

It is preferable to synthesize an administration model rather than develop an analysis-based model. In a synthesized model, the developer can build a process with preset properties as well as consider, in a formal view, the regularities that are common for the subject field [8].

The basis for building the concept should be a synthesized process management model (an inverse problem of management process). Burlov [9] identifies three main region's performance indicators::

$s_1(t)$ is a population size;

$s_2(t)$ is a number of jobs in real economy;

$s_3(t)$ is an indicator of energy availability in a region.

System-forming basis for the dynamic model is a system of differential equations (hereinafter - SDE) which describes the changes of the three major system-forming indicators. Regional policy implementation mechanisms are expressed through SDE factors for the benefit of achieving the government ad-ministration objective.

The conclusion of this system of differential equations can be drawn from the scientific and socio-economic interpretation of the meaning of the system-forming region's performance indicators.

2. A management model of a region's sustainable socio-economic development

The derivative of the region's population size ds_1/dt is a population size change rate, which is naturally connected both with the population size itself, where s_2 is a number of jobs in the real economy (economic performance indicator) and s_3 is an indicator of energy availability in the region. The change rate s_1 is proportional to the quantitative composition of the region, i.e. the larger the population, the larger its increase [10]:

$$\frac{ds_1}{dt} = u_1 s_1,$$

where u_1 is a demographic activity factor.

Let us define the influence of s_2 and s_3 on the population size change rate. The number of jobs in the region's real economy s_2 is determined based on the minimal number of workers at the enterprises needed for production of a certain range of goods and services. For a given value s_1, the number of filled jobs in the region's real economy s_2, will decrease the population size change rate by the value $u_2 s_1 s_2$, where u_2 is a childbirth demotivation factor.

For a given value s_1, the s_3 energy availability indicator will contribute to the increase in the population size change rate by the value $u_3 s_1 s_3$, where u_3 is a factor of energy availability in the region. In other words, the more energy in the region, the higher is the population size growth rate [9].

The first differential equation is rearranged as follows:

$$\frac{ds_1}{dt} = u_1 s_1 - u_2 s_1 s_2 + u_3 s_1 s_3.$$

The derivative of the number of jobs in real economy of the region ds_2/dt is the change rate of the economic development indicator. The change rate of the economic development indicator ds_2/dt decreases as long as the number of jobs in real economy is increasing, i.e. the more jobs are in the real economy, the harder it to increase the number of these jobs:

$$\frac{ds_2}{dt} = -u_4 s_2,$$

where u_4 is a factor of people's interest in development of the economy. For a given s_2 value, if people are interested in the development of the economy, the population size growth in the region, s_1, will change the real economy development rate, s_2, by the value $u_5 s_1 s_2$, where u_5 is the factor of the people's interest in the development of the economy. The manifestation of such a property is objective because it stems from the society's self-preservation instinct. For a given s_2 value, energy availability in the region, s_3, will contribute to the growth of the economic development rate, s_2, by the value $u_6 s_2 s_3$, where u_6 is the factor of energy availability in workplaces. I.e., the more energy is available for the development of real economy, the higher is the economic development growth rate (Burlov, V. 2007). The second differential equation reads:

$$\frac{ds_2}{dt} = -u_4 s_2 + u_5 s_1 s_2 + u_6 s_2 s_3.$$

The derivative of the region's energy availability indicator ds_3/dt is the change rate of energy availability in the region. The energy consumption change rate is proportional to the amount of the consumed energy, i.e. the more energy is consumed in the society, the higher is the rate of its growth:

$$\frac{ds_3}{dt} = u_7 s_3,$$

where u_7 is a factor of the energy availability development in the region.

For a given value s_3 the growth in the region's population size s_1 will decrease the energy consumption change rate by the value $u_8 s_1 s_3$, where u_8 is a factor of correspondence between the population and energy availability. Population growth decreases the energy availability change rate. For a given value s_3, an increase in a number of jobs in the real economy s_2 will contribute to a fall in energy availability change rate by the value $u_9 s_2 s_3$, where u_9 is a factor of correspondence between economic development and energy availability. In other words, the more the region's real economy is developed, the less energy there is for one workplace in real economy [9]. Therefore, the third differential equation reads:

$$\frac{ds_3}{dt} = u_7 s_3 - u_8 s_1 s_3 - u_9 s_2 s_3.$$

Thus, the management model of a region's sustainable socio-economic development would have the following form (all indicators are presented as relative, non-dimensional values):

$$\begin{aligned} \frac{ds_1(t)}{dt} &= u_1^{(0)} s_1(t) - u_2^{(0)} s_1(t) s_2(t) - u_3^{(0)} s_1(t) s_3(t), \\ \frac{ds_2(t)}{dt} &= -u_4^{(0)} s_2(t) + u_5^{(0)} s_1(t) s_2(t) + u_6^{(0)} s_2(t) s_3(t), \\ \frac{ds_3(t)}{dt} &= u_7^{(0)} s_3(t) - u_8^{(0)} s_1(t) s_3(t) - u_9^{(0)} s_2(t) s_3(t), \end{aligned} \tag{1}$$

wherein demographic activity factor,

$$s_1(t) = \frac{s_1^*(t_0) - s_1{}^*(t)}{s_1{}^*(t_0)},$$

$s_1^*(t)$ is the population size at time t; $s_1^*(t_0)$ is the population size at the initial time t_0; economic indicator:

$$s_2(t) = \frac{s_2{}^*(t_0) - s_2{}^*(t)}{s_2{}^*(t_0)},$$

$s_2^*(t)$ is the current number of jobs in real economy at the time t; $s_2^*(t_0)$ is a number of jobs in the real economy at the time t_0; energy availability indicator:

$$s_3(t) = \frac{s_3^*(t_0) - s_3^*(t)}{s_3^*(t_0)},$$

$s_3^*(t)$ is the current energy availability indicator at the time t; $s_3^*(t_0)$ is the energy availability

indicator at the time t_0;
u_1 is the demographic activity coefficient;
u_2 is the childbirth demotivation coefficient;
u_3 is the factor of energy availability in the region;
u_4 is the factor of people's interest in economic development;
u_5 is the factor of real economy development;
u_6 is the factor of energy availability at workplaces;
u_7 is the factor of energy availability development in a region;
u_8 is the factor of population conformity to energy availability;
u_9 is the factor of economic development conformity to energy availability.

3. Formalized concept of a regional management

The afore-mentioned nine factors, included to the system of differential equations, are components of a government administration vector, while the administration vector itself, for this particular model, has the following form:

$$u(t) = \left(u_1^0, u_2^0, u_3^0, u_4^0, u_5^0, u_6^0, u_7^0, u_8^0, u_9^0\right).$$

The vector of the region's condition for this model is the following one $s(t) = (s_1(t), s_2(t), s_3(t))$.

Such interpretation of the concept of government administration in a region helps to use the following problem of optimal management as the basis for the concept's implementation.

Here is the management problem, based on selection of the control function through a set of numbers, i.e. management per se becomes a function of parameters. It is necessary to define the vector of the region's condition, $s(t) = (s_1(t), s_2(t), s_3(t))$, as a solution of the system (1) with the control vector and boundary conditions:

$s_1(t)$ is the the region's population size is not defined (the indicator is vacant);

$s_2(t) = y^*$ is the fixed (a fixed number of jobs in the real economy is preset);
$s_3(t) \Rightarrow \min$.

Such a problem statement helps to solve the problem of government administration as a non-linear programming problem with the help of the proven methods [4, 11].

4. Numerical experiment

The standard approach to the analysis of a dynamic system (1) behavior is the study of its equilibrium points, which, in this case, are:

$$(0,0,0),\ \left(0, \frac{u_7}{u_9}, \frac{u_4}{u_6}\right),$$
$$\left(\frac{-u_4u_3u_9 - u_1u_6u_9 + u_2u_6u_7}{u_3u_5u_9 - u_2u_6u_8}, \frac{u_5u_3u_7 - u_4u_3u_8 - u_1u_6u_8}{u_3u_5u_9 - u_2u_6u_8}, \frac{-u_1u_5u_9 + u_2u_5u_7 - u_2u_4u_8}{u_3u_5u_9 - u_2u_6u_8}\right),$$
$$\left(\frac{u_7}{u_8}, 0, -\frac{u_1}{u_3}\right),\ \left(\frac{u_4}{u_5}, \frac{u_1}{u_2}, 0\right).$$

Taking into account the economic meaning accorded to s_1, s_2 and s_3, only the third point seems to be of interest. For the selected point, the Jacobi Matrix can be rewritten as follows:

$$D^{-1}\begin{pmatrix} 0 & u_2(-u_4u_3u_9-u_1u_6u_9+u_2u_6u_7) & u_3(u_4u_3u_9+u_1u_6u_9-u_2u_6u_7) \\ u_5(u_5u_3u_7-u_4u_3u_8-u_1u_6u_8) & 0 & u_6(u_5u_3u_7-u_4u_3u_8-u_1u_6u_8) \\ u_8(u_1u_5u_9-u_2u_5u_7+u_2u_4u_8) & u_9(u_1u_5u_9-u_2u_5u_7+u_2u_4u_8) & 0 \end{pmatrix},$$

where $D = u_3u_5u_9 - u_2u_6u_8$. Using the positivity of s_1, s_2, and s_3 in a chosen point, we have to impose additional conditions on parameters u_i:

$$\begin{cases} u_3u_5u_9 - u_2u_6u_8 \neq 0, \\ u_6 = -u_5 + u_4, \\ u_7 > \frac{u_1u_9+u_3u_9}{u_2}, \\ u_7 \neq \frac{u_4u_3u_9+u_1u_6u_9}{u_2u_6}, \\ u_8 = \frac{-u_1u_5u_9-u_5u_3u_9+u_2u_5u_7}{u_2(u_4-u_6)}. \end{cases} \quad (2)$$

Numerical experiments have been carried out for a set of parameters defined by the grid: they vary from from 1 to 10 with an increment equal to 1. The goal of the experiments have been to select the points where the system demonstrates stable behavior from the whole set of stationary points. Unfortunately, no such points, which would meet these conditions, have been found. But we have identified a subset of points where the system behavior is quite close to a stable one (in this particular case, these are the points for which the value of the real parts of the corresponding eigenvalues does not exceed 0.01). Figure 1 shows the distribution of the steady state points in the phase space

For all points shown in Fig. 1 $\Im\mathrm{m}\lambda_1 = 0$, $\Im\mathrm{m}\lambda_2 \neq 0$, $\Im\mathrm{m}\lambda_3 \neq 0$, as well as $\Re\mathrm{e}\lambda_1\Re\mathrm{e}\lambda_j < 0, j = 2, 3$.

It should be also noted that one and the same point in the space (s_1, s_2, s_3) can correspond to different sets of the parameters u_j, e.g. the point $(2, 1, 1)$ corresponds to the following sets of parameters $u_1 = 1$, $u_2 = 10$, $u_3 = 9$, $u_4 = 10$, $u_5 = 3$, $u_6 = 4$, $u_7 = 7$, $u_8 = 2$, $u_9 = 3$ and $u_1 = 2$, $u_2 = 9$, $u_3 = 7$, $u_4 = 9$, $u_5 = 4$, $u_6 = 1$, $u_7 = 7$, $u_8 = 3$, $u_9 = 1$.

In the figures 2 and 3, the examples of behavior near the point $(2.05, 2.95, 1.95)$ of the system (1) are shown. In this example, the parameters are $u = (0.1, 0.1, 0.1, 2.1, 0.1, 2.1, 8.1, 0.1, 4.1)$.

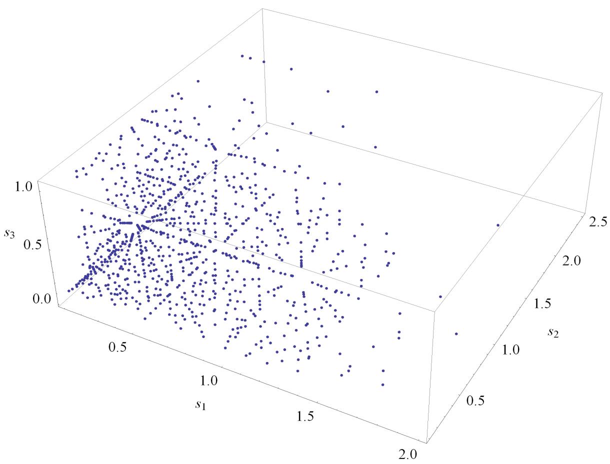

FIG. 1: The distribution of stationary points.

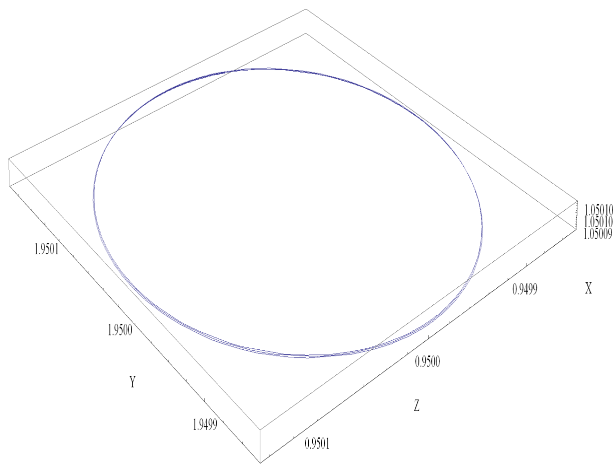

FIG. 2. The trajectory with initial perturbation $(0.0001, 0.0001, 0.0001)$ at the stable point $(2.05, 2.95, 1.95)$.

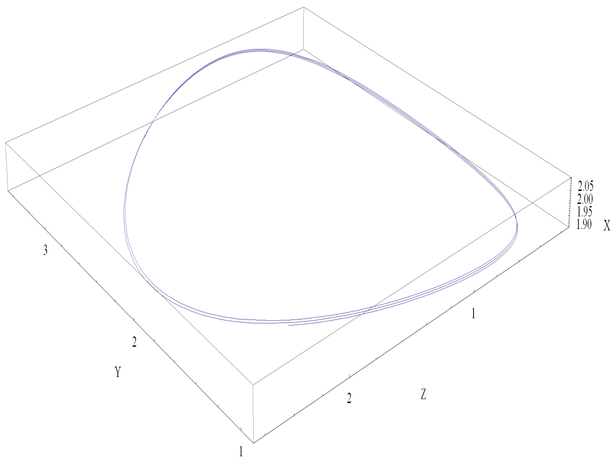

FIG. 3. The trajectory with initial perturbation (1, 1, 1) at the same point.

5. Conclusion

In the entire parameter space (within the system's parameter value restrictions, see the restrictions (2)), the study has failed to identify the points where the system demonstrates a stable behavior. The analysis has shown that a subset of points can be identified where the behavior of the system is close to the steady state.

A further research could target specific factor modifications that would help replace the present quasi-stable state of a region with the one we seek.

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